

COMMON PRE-BOARD EXAMINATIONS - 2023

Mathematics (Basic) (241)

MARKING SCHEME

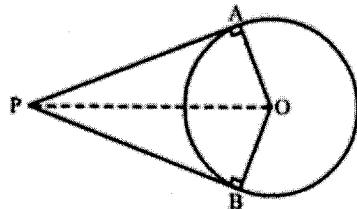
Maximum Marks: 80

Section A		
1.	(a) 7 yrs, 49 yrs	1
2.	(c) 20	1
3.	(a) 209	1
4.	(a) 3	1
5.	(b) ± 4	1
6.	(a) - 12	1
7.	(a) 15 cm	1
8.	(c) $75\sqrt{3}$	1
9.	(a) 25°	1
10.	(b) $\frac{1}{\sqrt{2}}$	1
11.	(a) 45°	1
12.	(a) $\frac{154}{3} \text{ cm}^2$	1
13.	(c) 32 cm	1
14.	(c) $240\pi \text{ cm}^3$	1
15.	(b) 0	1
16.	(c) 22 cm	1
17.	(d) 16 m	1
18.	(c) 3	1
19.	(d) Assertion (A) is false but Reason (R) is true.	1
20.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).	1

Section B

21. The required number of books is the LCM of 48 and 60.
 $48 = 2^4 \times 3$
 $60 = 2^2 \times 3 \times 5$
 $\text{LCM} = 2^4 \times 3 \times 5 = 16 \times 15 = 240$
Hence, required number of books is 240.

$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ \hline & 3 \end{array} \quad \begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ \hline & 5 \end{array}$$

22. Let AP and BP be the two tangents to the circle with centre O.
- 
- To Prove : $AP = BP$
- Proof :
- In $\triangle AOP$ and $\triangle BOP$
 $OA = OB$ (radii of the same circle)
 $\angle OAP = \angle OBP = 90^\circ$ (since tangent at any point of a circle is perpendicular to the radius through the point of contact)
 $OP = OP$ (common)
 $\therefore \triangle AOP \cong \triangle BOP$ (by R.H.S. congruence criterion)
 $\therefore AP = BP$ (corresponding parts of congruent triangles)
- Hence the length of the tangents drawn from an external point to a circle are equal.

- 23.
- Perimeter of quadrant = $2r + \frac{1}{4} \times 2\pi r$
 $\Rightarrow \text{Perimeter} = 2 \times 14 + \frac{1}{2} \times \frac{22}{7} \times 14$
 $\Rightarrow \text{Perimeter} = 28 + 22 = 50 \text{ cm}$

OR

$$\text{Area of first circle} = \pi r^2 = \pi(24)^2 = 576\pi \text{ m}^2$$

1/2

$$\text{Area of second circle} = \pi r^2 = \pi(7)^2 = 49\pi \text{ m}^2$$

Now, we are given that,

1/2

Area of the circle = Area of first circle + Area of second circle

$$\therefore \pi R^2 = 576\pi + 49\pi$$

(where, R is the radius of the new circle)

$$\Rightarrow \pi R^2 = 625\pi$$

$$\Rightarrow R^2 = 625$$

⇒ R = 25

\therefore Radius of the circle = 25cm

Thus, diameter of the circle = $2R = 50$ cm.

24.

In $\triangle OPQ$, we have

$$AB \parallel PQ$$

Therefore, by using basic proportionality theorem , we have

IN $\triangle OPR$, we have

AC || PR

Therefore, by using basic proportionality theorem , we have

Comparing (i)&(ii), we get

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Therefore, by using converse of basic proportionality theorem, we get

$$BC \parallel QR$$

OR

In $\triangle ABC$,

$ML \parallel BC$ (given)

$$\frac{AM}{AB} = \frac{AL}{LC} \dots\dots \text{(i) (By Basic Prop. Theorem)}$$

Again in $\triangle ADC$,

$LN \parallel DC$ (given)

$$\frac{AN}{ND} = \frac{AL}{LC} \dots\dots \text{(ii) (By Basic prop. Theorem)}$$

From equation (i) and (ii)

$$\frac{AM}{MB} = \frac{AN}{ND}$$

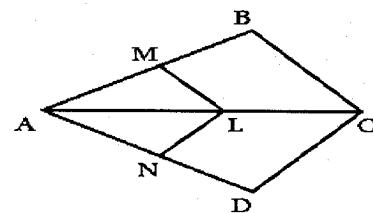
$$\text{or } \frac{MB}{AM} = \frac{ND}{AN}$$

$$\Rightarrow \frac{MB}{AM} + 1 = \frac{ND}{AN} + 1$$

$$\Rightarrow \frac{MB + AM}{AM} = \frac{ND + AN}{AN}$$

$$\Rightarrow \frac{AB}{AM} = \frac{AD}{AN}$$

$$\text{Thus, } \frac{AM}{AB} = \frac{AN}{AD}$$



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25.

(i) As per the question the bag contains lemon flavoured candies only. It does not contain even a single orange flavor candy.

1

$$P(\text{an orange flavoured candy}) = 0$$

(ii) As the bag has lemon flavoured candies, Malini will take out only lemon flavoured candies. Therefore, event that Malini will take out a lemon flavoured candy is a sure event

1

$$P(\text{a lemon flavoured candy}) = 1$$

Section C	
26.	<p>Let us assume that $3 + 2\sqrt{5}$ is a rational number. So, it can be written in the form $\frac{a}{b}$</p> $3 + 2\sqrt{5} = \frac{a}{b}$ <p>Here a and b are coprime numbers and $b \neq 0$</p> <p>Solving $3 + 2\sqrt{5} = \frac{a}{b}$ we get,</p> $\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3$ $\Rightarrow 2\sqrt{5} = \frac{a-3b}{b}$ $\Rightarrow \sqrt{5} = \frac{a-3b}{2b}$ <p>This shows $\frac{a-3b}{2b}$ is a rational number.</p> <p>But we know that $\sqrt{5}$ is an irrational number.</p> <p>So, it contradicts our assumption.</p> <p>Our assumption of $3 + 2\sqrt{5}$ is a rational number is incorrect.</p> <p>$3 + 2\sqrt{5}$ is an irrational number</p>
27.	<p>Let $f(x) = x^2 + 3x - 10$</p> <p>Put $f(x) = 0$</p> $x^2 + 3x - 10 = 0$ $x^2 + 5x - 2x - 10 = 0$ $x(x + 5) - 2(x + 5) = 0$ $(x + 5)(x - 2) = 0$ $\therefore x = -5 \text{ or } x = 2$ <p>Now, sum of zeroes = $-5 + 2 = -3 = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$</p> <p>Product of zeroes = $(-5) \times (2) = \frac{10}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$</p> <p>Hence, relationship verified.</p>
28.	<p>Let the ABCD be a rectangle in which shorter side of rectangle be $BC = x$ m.</p> <p>According to given condition, diagonal $= (x + 60)$ m and longer side $AB = (x + 30)$ m</p>

By applying Pythagoras theorem:

$$\text{Hypotenuse}^2 = \text{Side 1}^2 + \text{Side 2}^2$$

$$(60 + x)^2 = x^2 + (30 + x)^2$$

$$60^2 + 2(60)x + x^2 = x^2 + 30^2 + 2(30)x + x^2$$

$$3600 + 120x + x^2 = x^2 + 900 + 60x + x^2$$

$$3600 + 120x + x^2 - x^2 - 900 - 60x - x^2 = 0$$

$$2700 + 60x - x^2 = 0$$

Multiplying both sides by -1:

$$x^2 - 60x - 2700 = 0$$

$$b^2 - 4ac = (-60)^2 - 4(1)(-2700)$$

$$= 3600 + 10800$$

$$b^2 - 4ac = 14400 > 0$$

∴ Roots exist.

$$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$$

$$= [-(-60) \pm \sqrt{(14400)}] / 2$$

$$= [(60) \pm 120] / 2$$

$$x = (60 + 120) / 2 \text{ and } x = (60 - 120) / 2$$

$$x = 180 / 2 \text{ and } x = -60 / 2$$

$$x = 90 \text{ and } a = -30$$

Length can't be a negative value.

Hence, $x = 90$

Length of shorter side is $x = 90 \text{ m}$

Length of longer side = $30 + x = 30 + 90 = 120 \text{ m}$

OR

Let the 2 digits be x, y , respectively

$$\text{2 digit number} = 10x + y$$

$$\text{2 digit number obtained by ususing the digit} = 10y + x$$

$$\therefore 10x + y + 10y + x = 99$$

$$\Rightarrow 11x + 11y = 99$$

$$\Rightarrow x + y = 9$$

$$x - y = 3$$

$$\text{On adding : } 2x = 12$$

$$x = 6$$

$$\text{putting } x \text{ in } \rightarrow (i)$$

$$6 + y = 9$$

$$y = 3$$

$$\therefore \text{Number} = 10x + y = 63$$

$$\text{or , } 10y + x = 36.$$

29. $(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)$
 $(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)$
 $(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)$
 $(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)$
 $(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)$
 $(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)$

Total numbers = 36

(i) Sum of two numbers on the top is 8

i.e., $(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)$

\therefore Number of favourable outcomes = 5

$$\therefore P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

$$= \frac{5}{36}$$

(ii) Sum of two number on the top is 13.

At the most, then sum can be $(6, 6) = 12$

\therefore number of favourable outcomes = 0

$$\therefore P(E) = 0$$

(iii) Sum is less than or equal to 12
 \therefore Number of favourable outcomes = 36

$$\therefore P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

$$= \frac{36}{36} = 1 \text{ Ans.}$$

30. Given: XY is a tangent at point P
 and X'Y' is a tangent at point Q.
 And XY || X'Y'
 AB is a tangent at point C

To prove: $\angle AOB = 90^\circ$

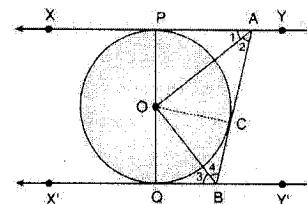
Proof: Join OC

For tangent AB & Radius OC

$OC \perp AB$

(Tangent at any point of circle is perpendicular to the radius through point of contact)

So, $\angle ACO = \angle BCO = 90^\circ$



In $\triangle OPA$ and $\triangle OCA$

$OP = OC$ (radii of same circle)

$PA = CA$ (length of two tangents from an external point)

$OA = OA$ (Common side)

$$\therefore \triangle OPA \cong \triangle OCA$$

(By SSS congruency criterion)

$$\text{Hence, } \angle 1 = \angle 2 \quad (\text{cpct})$$

$$\text{Similarly } \angle 3 = \angle 4$$

Now,

$$\angle PAB + \angle QBA = 180^\circ$$

$$\Rightarrow 2\angle 2 + 2\angle 4 = 180^\circ$$

$$\Rightarrow \angle 2 + \angle 4 = 90^\circ$$

$$\Rightarrow \angle AOB = 90^\circ$$

(Angle sum property)

31.

$$\sin\theta + \cos\theta = \sqrt{3} \quad \dots\dots(Given)$$

Squaring, we get

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$

$$1 + 2\sin\theta\cos\theta = 3$$

$$2\sin\theta\cos\theta = 2$$

$$\sin\theta\cos\theta = 1 \quad \dots\dots(1)$$

Now, $\tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{1}{1} \quad \dots\dots[From(1)]$$

$$= 1$$

OR

$$= \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\tan\theta}{1 - \tan\theta} \left(\because \tan\theta = \frac{1}{\cot\theta} \right)$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{(1 - \tan\theta)\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta}$$

$$[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)(\tan\theta)} = \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$$

$$= \tan\theta + 1 + \cot\theta$$

$$= \text{RHS.}$$

Hence proved.

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Section D

32.

$$\begin{aligned}x - y + 1 &= 0 \\x &= y - 1\end{aligned}$$

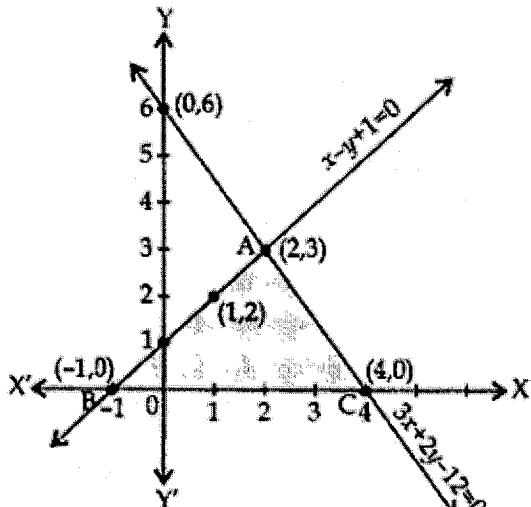
x	-1	1	2
y	0	2	3

(-1, 0), (1, 2), (2, 3)

$$\begin{aligned}3x + 2y - 12 &= 0 \\2y &= 12 - 3x \\y &= \frac{12 - 3x}{2}\end{aligned}$$

x	0	2	4
y	6	3	0

(0, 6), (2, 3), (4, 0)



Lines intersect at (2, 3)

$$\therefore x = 2, y = 3$$

Vertices of $\triangle ABC$ are A(2, 3), B(-1, 0) and C(4, 0)

OR

Let the speed of car at A be x kmph
and the speed of car at B be y kmph

when the car travel in same direction Relative Speed is $x - y$

Dist = 100km

t = 5 hours

\therefore Dist = S \times T

$$100 = (x - y)5$$

$$x - y = 20 \rightarrow (I)$$

when car travel in opp direction Relative Speed is $x + y$

Dist = 100km

t = 1 hours

Dist = ST

$$100 = (x + y)1$$

$$x + y = 100 \rightarrow (II)$$

Solving (I) & (II)

$$x - y = 20$$

$$\underline{x + y = 100}$$

$$2x = 120$$

km/h

$$x = 60\text{ km/h}$$

$$y = 40\text{ km/h}$$

Speed of the car at A = 60 km/h

Speed of the car at B = 40 km/h

The difference of speeds is $60 - 40 = 20$

33.

Class interval:	Frequency: (f_i)	Cumulative frequency (c.f.)
0-10	f_1	f_1
10-20	5	$5 + f_1$
20-30	9	$14 + f_1$
30-40	12	$26 + f_1$
40-50	f_2	$26 + f_1 + f_2$
50-60	3	$29 + f_1 + f_2$
60-70	2	$31 + f_1 + f_2$
	$N = 40 = 31 + f_1 + f_2$	

Now, we have

$$N = 40$$

$$31 + f_1 + f_2 = 40$$

$$f_2 = 9 - f_1 \dots\dots(1)$$

$$\text{Also, } \frac{N}{2} = 20$$

Since median = 32.5 so the median class is 30-40.

Here, $I = 30$, $f = 12$, $F = 14 + f_1$ and $h = 10$

We know that

$$\text{Median} = l + \left\{ \frac{\frac{N}{2} - F}{f} \right\} \times h$$

$$32.5 = 30 + \left\{ \frac{20 - (14 + f_1)}{12} \right\} \times 10$$

$$2.5 = \frac{(6 - f_1) \times 10}{12}$$

$$2.5 \times 12 = 60 - 10f_1$$

$$10f_1 = 60 - 30$$

$$f_1 = \frac{30}{10} \\ = 3$$

Putting the value of f_1 in (1), we get

$$f_2 = 9 - 3 \\ = 6$$

Hence, the missing frequencies are 3 and 6.

34.

Since, the inner diameter of the glass = 5 cm and height = 10 cm,
the apparent capacity of the glass = $\pi r^2 h$

$$= (3.14 \times 2.5 \times 2.5 \times 10) \text{ cm}^3 = 196.25 \text{ cm}^3$$

But the actual capacity of the glass is less by the volume of the hemisphere at the base of the glass.

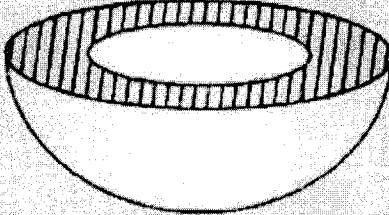
$$\text{i.e., it is less by } \frac{2}{3} \pi r^3 = \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5 \text{ cm}^3 \\ = 32.71 \text{ cm}^3$$

So, the actual capacity of the glass

$$= \text{Apparent capacity of glass} - \text{Volume of the hemisphere} \\ = (196.25 - 32.71) \text{ cm}^3 = 163.54 \text{ cm}^3$$

OR

$$R = 8 \text{ cm}, r = 6 \text{ cm}$$



$$\begin{aligned}\text{Surface area} &= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2) \\&= \pi[8^2 \times 2 + 6^2 \times 2 + (8^2 - 6^2)] \\&= \pi[64 \times 2 + 36 \times 2 + (64 - 36)] \\&= \pi[128 + 72 + 28] \\&= 228 \times 3.14 \\&= 715.92 \text{ cm}^2\end{aligned}$$

$$\text{Total cost} = 715.92 \times 5 = \text{Rs. } 3579.60.$$

35. Given : a triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

To prove : $AD/ AE = DB /EC$.

Construction : Join BE and CD and then draw $DM \perp AC$ and $EN \perp AB$.

Proof : Now, area of ΔADE ($1/2$ base \times height) $= 1/2 AD \times EN$.

$$\text{So, } \text{ar}(ADE) = 1/2 \times AD \times EN$$

$$\text{Similarly, } \text{ar}(BDE) = 1/2 \times DB \times EN,$$

$$\text{ar}(ADE) = 1/2 \times AE \times DM$$

$$\text{and } \text{ar}(DEC) = 1/2 \times EC \times DM.$$

$$\text{Therefore, } \text{ar}(ADE)/ \text{ar}(BDE) = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad \dots \dots \dots (1)$$

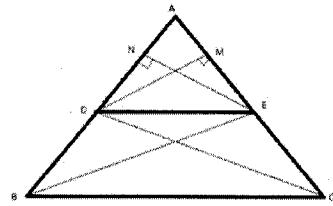
$$\text{ar}(ADE) / \text{ar}(DEC) = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \dots \dots \dots (2)$$

Note that ΔBDE and ΔDEC are on the same base DE and between the same parallels BC and DE.

$$\text{So, } \text{ar}(BDE) = \text{ar}(DEC) \quad \dots \dots \dots (3)$$

Therefore, from (1), (2) and (3), we have:

$$AD/ DB = AE/ EC$$



Section E		
Case Study - 1		
36.	<p>A (3, 4), B(6, 7), C(9, 4), D(6, 1)</p> <p>(i) the distance between A and B = $\sqrt{(6-3)^2 + (7-4)^2} = \sqrt{9+9} = 3\sqrt{2}$</p> <p>(ii) the distance between C and D = $\sqrt{(6-3)^2 + (7-4)^2} = \sqrt{9+9} = 3\sqrt{2}$</p> <p>(iii) Let the point (4, 5) divides the line segment AB in the ratio k : 1.</p> $(4, 5) = \left(\frac{6k+3}{k+1}, \frac{7k+4}{k+1}\right)$ $4 = \frac{6k+3}{k+1}$ $4k+4 = 6k+3$ $k = \frac{1}{2}$ $K : 1 = 1 : 2$	1 1 ½ ½ ½ ½ ½ ½
	OR	
	<p>The position of E is the mid point AC and BD.</p> <p>By using mid point formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$</p> <p>Position of E = $\left(\frac{3+9}{2}, \frac{4+4}{2}\right) = (6, 4)$</p>	½ ½ ½ ½
Case Study - 2		
37.	<p>(i) AP is 51, 49, 47,</p> <p>(ii) Here, d = -2, a = 51 and nth term = 31</p> <p>nth term of an AP = a + (n - 1)d</p> $\Rightarrow 31 = 51 + (n - 1)(-2)$ $\Rightarrow 31 = -2n + 2 + 51$ $\Rightarrow 31 = -2n + 53$ $\Rightarrow -2n = 31 - 53 = -22$ $\Rightarrow n = 11$	1 ½ ½ ½ ½ ½
	OR	

$$S_n = \frac{n}{2} (a + l)$$

$$= \frac{11}{2} (51 + 31)$$

$$= 451 \text{ seconds.}$$

1

1

(iii) Given $2x, x+10$ and $3x+2$ are in A.P.

Then we'll have,

$$2(x+10) = 2x + 3x + 2$$

$$2x + 20 = 5x + 2$$

$$3x = 18$$

$$x = 6.$$

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Case Study - 3

38.

(i)

Let $AB = h$, $CB = x$ and $CD = 20$

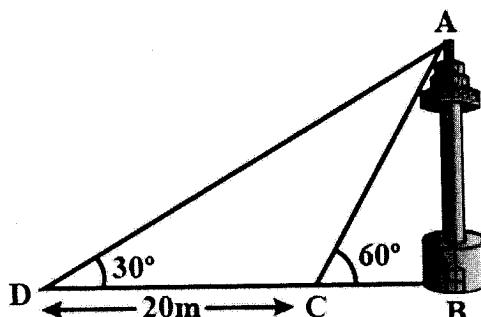
From right $\triangle BAC$.

$$\cot 60^\circ = x/h$$

$$1/\sqrt{3} = x/h$$

$$x = h/\sqrt{3} \dots (1)$$

$$h = x\sqrt{3} \text{ m}$$



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From right $\triangle BAD$:

$$\cot 30^\circ = AD/AB$$

$$\sqrt{3} = (x+20)/h$$

$$x = h\sqrt{3} - 20 \dots (2)$$

$$h = 10\sqrt{3}$$

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OR

From right $\triangle BAD$:

$$\cot 30^\circ = AD/AB$$

$\frac{1}{2}$

$$\sqrt{3} = (x + 20)/h$$

$\frac{1}{2}$

$$x = h\sqrt{3} - 20 \dots (2)$$

$$h = 10\sqrt{3}$$

$\frac{1}{2}$

$$\text{From (1): } x = (10\sqrt{3})/\sqrt{3} = 10$$

$\frac{1}{2}$

Width of the canal = BC = 10 m

(ii) In $\triangle ABC$, $\sin 60^\circ = \frac{AB}{BC}$

$$\frac{\sqrt{3}}{2} = \frac{10\sqrt{3}}{AC}$$

$\frac{1}{2}$

$$AC = 20 \text{ m}$$

$\frac{1}{2}$

(iii) In $\triangle ABD$, $\angle DCA = 180^\circ - 60^\circ = 120^\circ$

$\frac{1}{2}$

$$30^\circ + 120^\circ + \angle DCA = 180^\circ$$

$$\angle DAC = 180^\circ - 150^\circ = 30^\circ$$

$\frac{1}{2}$